TRINITY COLLEGE

Year 12 Mathematics Specialist 3/4
Test 1 2022

Section 1 Calculator Free

Complex Numbers

STUDENT'S NAME _____

DATE: Monday 28 February **TIME:** 20 minutes **MARKS**: 19

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

The function $f(x) = x^4 - x^3 + ax^2 + bx - 18$ has a root at $x = -2$ and has a remainder of -20 when divided by $(x-1)$.

Determine the values of a and b where $a, b \in \mathbb{Z}$.

2. (7 marks)

Consider the polynomial function $f(z) = z^4 + 7z^2 + 12$

(a) Show that $z - 2i$ is a factor of $f(z)$ [2]

(b) State another factor of $f(z)$ [1]

(c) Hence, or otherwise, solve $f(z) = 0$ [4]

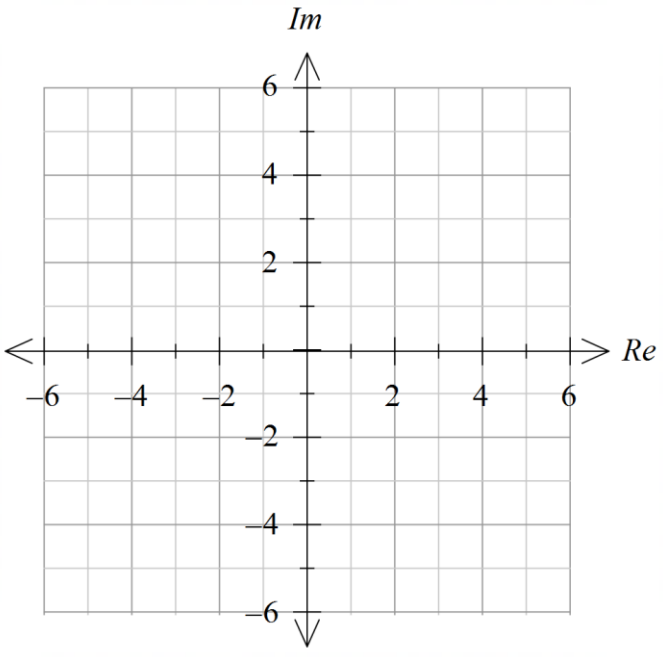
3. (8 marks)

Consider the locus of points defined for $\left\{ z : z \in \mathbb{C}, \frac{\pi}{2} \le \arg\left(z^2\right) < \pi \right\}$

(a) Show that $z = 2 + i$ is **not** in the locus. Explain. [3]

(b) Show that $z = 1 + i$ is in the locus. [2]

(c) On the Argand plane below, sketch the locus of points of z. [3]



TRINITY COLLEGE

Year 12 Mathematics Specialist 3/4
Test 1 2022

Section 2 Calculator Assumed

Complex Numbers

STUDENT'S NAME _____

DATE: Monday 28 February **TIME:** 30 minutes **MARKS**: 31

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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4. (6 marks)

Given the complex numbers $w = 2 - 2i$ and $u = 3cis\frac{\pi}{4}$, determine:

(a) $\arg\left(\frac{2i - \bar{w}}{u}\right)$ [3]

(b) $\left|u^2 w^2\right|$ [3]

5. (7 marks)

Consider the complex equation $z^5 + 16\sqrt{3} - 16i = 0$.

(a) Solve the equation giving exact solutions in the form $r\,cis\,\theta$ where $-\pi < \theta \leq \pi$. [4]

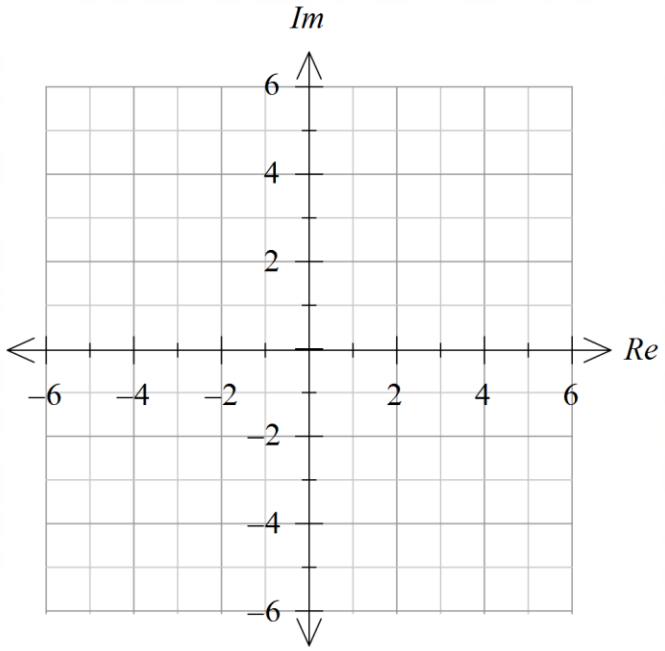
Let w be the solution to $z^5 + 16\sqrt{3} - 16i = 0$ with the greatest argument.

(b) Determine the exact value for $\arg(w-2)$ [3]

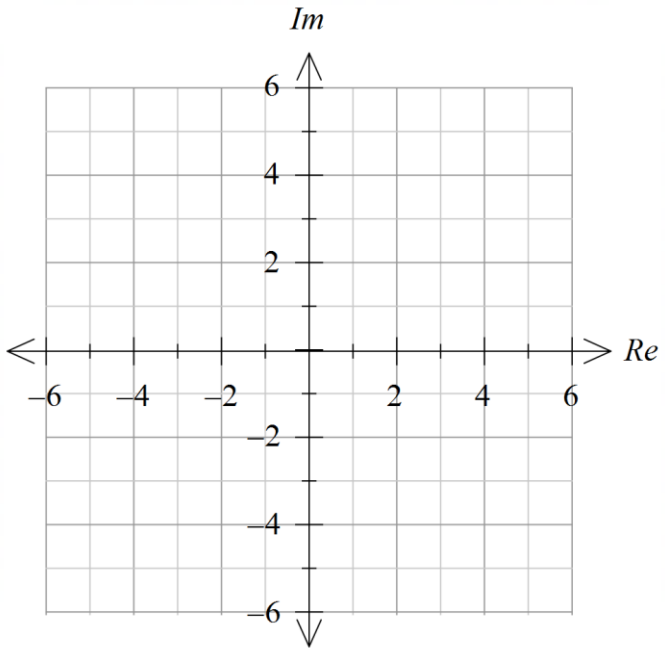
6. (10 marks)

(a) On the Argand planes below, sketch the locus of the complex number $z = x + iy$ given by:

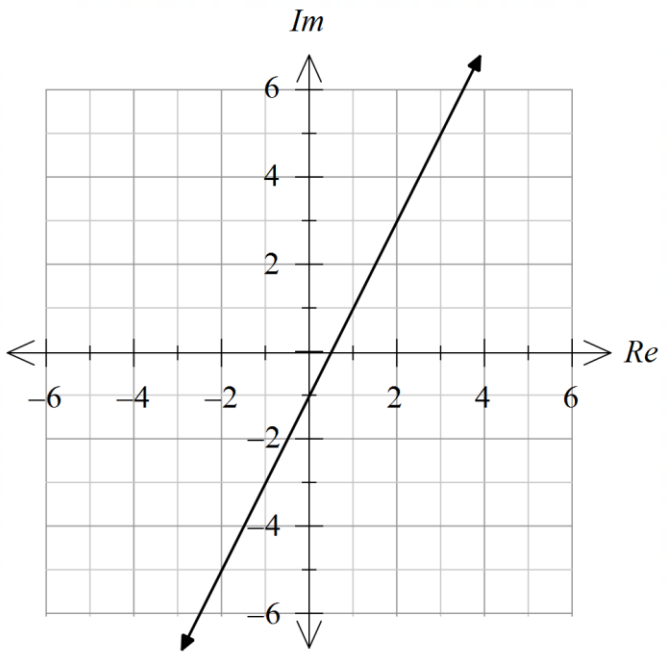
(i) $\{z : z \in \mathbb{C}, |z - 2 + i| > 2\}$ [3]



(ii) $\{z : z \in \mathbb{C}, |z - 3| + |z + 4i| = 5\}$ [3]



(b) A sketch of the locus of a complex number $z = x + iy$ is shown below.



(i) The equation is of the form $|z - w| = |z - 3|$ where $w \in \mathbb{C}$. Determine the value of w. [1]

(ii) Determine the minimum value for $|z + 4 - i|$ as an exact value. [3]

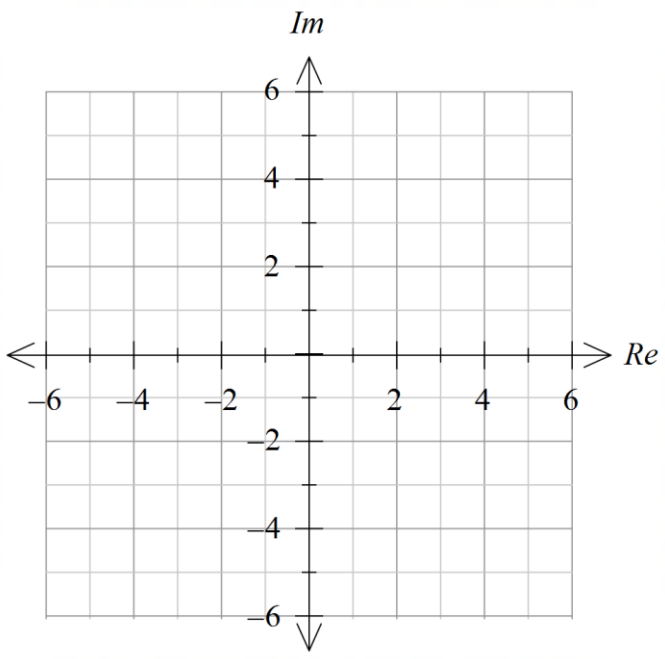
7. (8 marks)

Consider the locus of the complex number $z = x + iy$ given by $(1-i)z + (1+i)\overline{z} = 4$.

(a) Show that $z = 2i$ is in the locus. [2]

The locus forms a line.

(b) Hence, or otherwise, sketch the locus on the Argand diagram below. [2]



Let $z_1 = 2 + 2i$ be a point in the complex plane

(c) If the reflection of z_1 about the line in part (b) is z_2, calculate the value of $\overline{z}_1(1+i) + z_2(1-i)$. [4]